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Bootstrapping Sequential Tests for Multiple Structural Breaks

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Abstract

We show how finite sample bootstrapping methods can help to detect multiple breaks in systems of equations with long time series. The method of Banerjee and Urga (1995, 1996), where *single* breaks in the marginal models are imposed in the conditional model and then the conditional model estimated, is extended to cover the case of multiple (≥ 2) breaks in marginal and conditional models by using the technique of dominant break dating. An empirical investigation of a small monetary system for the United Kingdom establishes the viability of our method in developing congruent dynamic regression models.

Keywords: Sequential, Bootstrapping, Structural Breaks
JEL Classification Numbers: C10, C12, C13, C15

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1. Introduction

In recent work we have established methods for dating break dates sequentially. Our research is squarely within the family of papers published recently (Bai (1997), Bai and Perron (1998), and Culver and Papell (1997)) which has extended Perron's (1989) analysis not only to the case where the break date is unknown but to a scenario where the series may be broken, both in trend and in mean, more than once. To our mind, one of the main difficulties posed by this literature is the specificity of the applicable critical values to the particular data generation process (DGP) and to model combinations used. Papers by MacKinnon (1994) *inter alia* have emphasised the usefulness of response surfaces which may be used to re-compute critical values under changes to the DGP such as sample size, signal-noise ratio, unconditional mean etc. In this illustrative note, we propose the use of bootstrapping methods as another way of overcoming the difficulty of obtaining the applicable critical values. [2]

Centrally, our philosophy is to start with the largest available sample and estimate marginal models such as (1b) below, allowing for a *single* break in the mean and trend (simultaneously). From these equations, by sequential search, we thus first obtain a break date for the *full sample* using methods developed in Banerjee *et al.* (1992), Christiano (1992), Zivot and Andrews (1992) and Bai (1997). The "sequentiality" follows from allowing the location of the break to vary freely across this sample, subject only to trimming restrictions. We then bootstrap these equations n times on the full sample and record the density of the coefficient estimates on the break dates. This information is useful in enabling us to judge significance, at any conventional level, of the breaks.

Subsequent steps of the procedure take the form either of imposing the break dates for each of the models found at the first stage and repeating the exercise on the full sample, or of partitioning the sample at the break dates and implementing the procedure on the sub-samples thereby created. By allowing at each stage for a single break under the alternative, and iterating, we make use of the result (see, for example, Bai (1997)) that under-parameterisation of the number of breaks in the DGP biases the finding of the breaks towards the most dominant one in the data sample. Thus by controlling for the dominant breaks

[2] A valuable account of the usefulness of bootstrap methods in time series econometrics has appeared in a recent issue of *Econometric Reviews* (1996) and provides useful background for the discussion in our paper. There is an extensive literature on bootstrapping both in statistics and econometrics. In addition to the journal cited above, see, for example, Efron and Tibshirani (1993) (and references therein), Freedman and Peters (1984a), Freedman and Peters (1984b) and Basawa, Mallik, McCormick, Reeves and Taylor (1991).

one by one up to, say, stage k, allows us to pick the next dominant one at stage $k+1$.

The final stage of the procedure is to impose the break dates from the marginal models (1b) into the conditional model given by (1a) and repeat the bootstrapping exercise for the conditional model. What emerges at the end are congruent marginal and conditional models with the breaks in all the series and relations of interest properly identified.

A few remarks are necessary concerning the efficacy of the bootstrapping methods in the context of our investigations here. First, in our experience, especially with simulated samples, the dominant break is fairly unambiguously evident. When instead neighbouring breaks are found to be nearly as dominant as the main one, we take this to represent a gradual evolution of the mean or trend from one level to another, instead of an abrupt and discrete change. In principle we might easily attach weights to the dummies to represent this notion of gradual evolution more directly although we do not do so here.

Second, as noted above, output from the bootstrapping exercise, in addition to the central estimates of the coefficients in the model, also provides 95% or 99% confidence intervals (depending upon the width of the interval required) of these estimates. Depending on the nature of the data and the time-period under study, some of these confidence intervals may be quite precise while, a problem often generic to such cases, in others they may be too wide for detailed use to be made of the information. Nevertheless the confidence intervals do provide an important indication of the degree of "structural instability" in the data.

Lastly, we feel that the bootstrapping methods proposed here provide a useful way of looking at the case of multiple breaks. So far, our results are heuristic - we do not, for example, have detailed proofs of the statistical consistency of our methods - but the results in the particular examples we have looked at are so convincing that we have good reason to believe that our methods can be put to good use and justified rigorously. Moreover, by using finite sample methods and bootstrapping, the results can be generalised, in principle, to more complex models.[3]

[3] In preliminary Monte Carlo simulations we have tried to establish the power of bootstrapping methods such as the ones proposed here to detect structural instability. On artificially generated data, for which the DGP is known to us, the results are indeed encouraging and enable us to have faith in the empirical implementations reported below. Details of these experiments are also available from us.

We take as our main motivating empirical example a consideration of money data for OECD countries. In particular we focus on the age-old controversial issue of the constancy of velocity of money and propose that empirical and finite-sample analysis of this variable provide a useful test-bed for our methods. Recent papers by Hoffman, Rasche and Tieslau (1995) and Hoffman and Rasche (1996) have argued forcefully the Friedmanian case for velocity being one of the economic system's great constants. They have further shown that it is reasonably easy to obtain well-behaved equations for OECD countries, which show stability of velocity of money, once certain simplistic corrections have been made. However, the conclusions reached by them run contrary to much established research in this area. This has suggested instead that various structural changes in the monetary regime, such as credit liberalisation, the introduction of new interest bearing instruments and other monetary innovations, have an inherently destabilising effect on variables such as the velocity for money. Especially when the time period of the study, as in Hoffman and Rasche, is fairly lengthy.

In related detailed empirical work, one of us has shown that the Hoffman and Rasche results are based largely on misspecification of the vector autoregression (VAR) used in their paper (see Caporale *et al*. (1997)). The current note is supportive of this latter claim.

In summary, our paper serves two important purposes. It illustrates the main methods we propose for conducting structural break dating (as in many other exercises, our methods are meant to be used complementarily with other existing results) taking very simple DGP's. Secondly, it makes use of our methods to provide, we hope, powerful counter-argument to the line of thought that proposes that the behaviour of velocity is essentially characterised by a stationary (or slowly changing) process.

As part of a larger issue it will be important to determine whether what passes for structural instability in our methods are merely proxies for important omitted variables and whether their inclusion will lead to much of the instability being mopped up. There is some evidence in favour of the latter happening, when we proceed from the modelling of the marginal models to looking at the conditional model. There are also some grounds for regarding this progression from the marginal to the conditional as providing evidence for co-breaking in the sense of Hendry (1996).

In Section 2, two main methods of break dating based on sequential search and bootstrapping are proposed. Next we illustrate our methods using a small system consisting of money demand, income and interest rate for the

United Kingdom (UK). This is done in two stages. First, in Section 3, we use our methods to detect breaks in the so-called "marginal" processes for income and interest rate for the UK. Second, Section 4 uses this knowledge to specify empirical models for these processes that satisfy a range of model-specification criteria. The break dates identified, and tabulated in sequential form in Tables 1-2, may in some sense be regarded as too numerous. But this is only a problem to begin with. Proceeding from the general specification and simplifying eventually lead to much more manageable set-ups which can then be utilised to derive stable and interpretable marginal models which pass most reasonable dynamic specification tests. Imposing these breaks in the conditional model, and repeating the exercise for the conditional model then leads to stable conditional models with easily interpretable coefficients. These are reported in Section 5. Section 6 provides an interpretation of our empirical findings and concludes.

2. The Main Methods

As in our earlier work (Banerjee and Urga (1995, 1996)), we use a very simple system for illustration. Our justification for doing so is two-fold. It keeps the analysis simple and at the same time provides easily interpretable results.
Consider estimating the following system:

$$y_t = \mu_0 + \delta_0 t + \rho_0 y_{t-1} + a_1 x_{1,t-1} + a_2 x_{2,t-1} + \alpha_0 D_{0t} + \beta_0 DT_{0t} + \eta_1 D_{1t} + v_1 DT_{1t} + \eta_2 D_{2t} + v_2 DT_{2t} + u_t \tag{1a}$$

$$x_{i,t} = \mu_i + \delta_i t + \rho_i x_{i,t-1} + \alpha_i D_{it} + \beta_i DT_{it} + e_{it}, \quad i = 1,2 \tag{1b}$$

where x_{it} is a variable denoting either of the marginal processes, here taken to be income and interest rate. The dummy variables are defined as follows:

$$D_{st} = 1, \qquad t \geq T_s,\ s = 0,\ 1,\ 2;$$

$$DT_{st} = t - T_s + 1, \qquad t \geq T_s,\ s = 0,\ 1,\ 2. \tag{2}$$

The structure and notation utilised in the system is deliberate and reflects the fact that the dummies in the marginal models are imposed in the reduced form of the conditional model, following estimation of the location of the break in the marginal models using the procedure outlined below (and discussed in greater detail in our previous work). So that, for example, the break dummies in the marginal model appear with *different* coefficients in the conditional model. Following estimation of the marginal models, the conditional model with its own breaks is then estimated. Two alternative (but related) methods of break dating are now described.

2.1. Including the Breaks One by One

Under this method, as a first step in looking for breaks in a marginal process we run a sequential test procedure. We estimate the following regressions:

$$x_t = \mu + \delta t + \rho x_{t-1} + \alpha D_t + \beta DT_t + \varepsilon_t, \qquad (3)$$

where
$$D_t = DT_t = 0 \qquad t = 1,2,\ldots,t_0 - 1$$
$$D_t = 1,\ DT_t = t - t_0 + 1 \qquad t = t_0,\ldots,T,$$

for $t_0 = 3,\ldots,T-1$ and $t = 2,\ldots,T$, using the first observation x_1 as an initial condition. For each t_0, the F-statistic for testing the null hypothesis of no break, $\alpha = \beta = 0$, was computed. A break-point estimator is defined as the t_0 at which the F-statistics attains its maximum.

In the next step, we include the two dummies D_{t_0} and DT_{t_0} into the regressions and repeat the sequential F-test procedure to obtain another break, skipping the date of included break. We continue in this manner until we find, say, m breaks.

Ideally, we would like to stop when the last estimated break is not significant, that is when the null of m breaks against the alternative of $m+1$ breaks is not rejected. The conventional test of this hypothesis would be to compare the computed F-statistic against the 5% critical value of the relevant F-distribution. However, this is not appropriate in our case as the time of break is not arbitrated exogenously. Specifically, the standard F-distribution critical values appear to be too low, causing excessive rejection of the null hypothesis of structural stability.

To obtain a critical value that takes into account the structure of our model, we bootstrapped the F-values adopting two different approaches. In the first approach we take into account the fact that the break was chosen endogenously on the basis of the maximum F-value from the sequential procedure. The regression (3) with imposed alternative of one more break at time t_0 is run, the residuals are re-sampled and the estimated parameters together with re-sampled residuals are used to create a recursive bootstrap sample following the equation (3). For 1000 generated bootstrap samples a regression (3) is estimated and the F-test of the hypothesis $\alpha = \beta = 0$ is carried out. The empirical F-distribution obtained in this manner is used to get a 5% critical value for the F-statistic for the time of break fixed at t_0. This critical value turned out to be too conservative, bringing about a huge loss of power. Christiano (1992) came to the same conclusion in a similar case of search for breaks.

In the second, we ignore the pre-test examination of data. This amounts to estimating the regressions under the null, where the dummies for the last break are not included. The date of break is chosen ex-post on the basis of p-values computed from the bootstrap distribution of F-values for each year. This procedure follows closely Christiano (1992). The critical values in this case are far less conservative than in the previous case, indeed the additional breaks do not show any tendency to become less significant than the preceding ones.

Finally, in an alternative attempt to gauge the significance of the last break, we bootstrapped the values of the coefficient estimates, employing again the regression under the alternative hypothesis. Here once more a test based on a two-tailed 95% confidence interval constructed from the bootstrapped coefficient distribution showed a tendency to over-reject under the null.

Thus with the sets of critical values being either too conservative or too radical, we do not have a convenient and unambiguous stopping rule based on significance tests. As a working rule, we choose the break on the maximum F-value basis and stop adding breaks when the coefficients in the regression pass tests for congruent specification.

2.2. Splitting the Sample

Under this method, we first run the sequential F-test for the regressions (3). We compute the F-statistics for all $t_0 = 3,\ldots,T-1$. Then we run the following regressions:

$$x_t = \mu + \delta t + \rho x_{t-1} + \varepsilon_t^0, \qquad (4)$$

for $t = 2,...,T$, using the first observation as an initial condition. We re-sample the residuals from this regression and generate a bootstrap sample x_t^* using the equation (4) with estimated parameters:

$$x_t^* = \hat{\mu} + \hat{\delta} t + \hat{\rho} x_{t-1} + \varepsilon_t^*,$$

where ε_t^* denotes the re-sampled ε_t^0 residuals. For the bootstrap sample, we run the sequential test for the regressions (3). We repeat re-sampling 1000 times, storing values of *F*-statistic for each time t_0 and for each bootstrap replication. The fiftieth highest value of the *F*-statistic for each date of break is then a 5% bootstrapped critical value for the *F*-statistic for that year. Further, the percentage of bootstrapped *F*-values exceeding the empirical *F*-value is the relevant *p*-value. The break-point estimator is now the date with the minimal *p*-value. We call this break dominant if the peak in the *p*-value graph is sharp.

Our next step is to repeat the bootstrapping procedure in each of the two sub-samples created by splitting the sample at the estimated break point and to continue this procedure until the dominance disappears or if the sub-sample is too short.[4]

3. Empirical Applications

3.1 Bootstrapped Estimates of the Breaks: Including the Breaks in the Marginal Models One by One.

The results obtained by applying the method of including the breaks in the marginal models one by one, are most conveniently depicted in Tables 1A and 1B below for both marginal processes, i.e. real income and interest rate for U.K respectively.

Our results are all based on 1000 bootstrap replications. The dominant breaks for each run, for each of the two marginal series considered, are tabulated below. Starting, for example, in Table 1A, with only one dummy for

[4] As a general rule, when considering estimating in partitioned sub-samples, we do not estimate sub-samples that are shorter than 8 -12 quarters in length. This is for two reasons. First we would run into degrees of freedom difficulties; second there would be reason to believe that any breaks found within such a short time span might in fact be reflective of the same break and repeated partitioning would prove to be misleading in its finesse.

break in mean and trend (step 1), the procedure utilises the maximum *F*-value method outlined in Section 2.1 to augment the model with the addition of further breaks at each step. We repeated the procedure until stable marginal models reported in Tables 3A and 3B below were successfully estimated. For both processes, five steps were required to achieve stability.

3.2 Bootstrapped Estimates of the Breaks: Splitting the Sample.

This sub-section reports the results of the method outlined in Section 2.2. The sub-sample partitions are given below and the results are also presented graphically in Figures 1-2.

It will be noted from comparing Table 1A-1B with Table 2A-2B, and the accompanying graphs, of the remarkable degree of congruence between the two break dating methods. Moreover, the results are very intuitively interpretable. In this regard, it is very instructive to look particularly at the results of this exercise for UK real income. In summary, five clear episodes of instability are identified, the period leading to and beyond the Oil Shock in the early 1970's, the recession just as Margaret Thatcher came into office in the late seventies, the 1983 recession, the 1987 stock market disturbance and the down-turn in the early 1990's.

Table 1A:
UNITED KINGDOM - Marginal 1 (log of real income)

	Step 1			Step 2			Step 3			Step 4			Step 5		
	Est. value	Bootstrapped 95% confidence interval		Est. value	Bootstrapped 95% confidence interval		Est. value	Bootstrapped 95% confidence interval		Est. value	Bootstrapped 95% confidence interval		Est. value	Bootstrapped 95% confidence interval	
Constant	0.370	0.249	1.426	0.405	0.315	1.271	0.428	0.260	1.389	0.965	0.622	1.591	0.988	0.609	1.759
Trend	0.002	0.001	0.004	0.001	-0.001	0.003	0.001	-0.001	0.003	0.002	0.000	0.004	0.002	-0.000	0.003
AR(1)	0.936	0.706	0.953	0.931	0.756	0.945	0.927	0.740	0.952	0.835	0.685	0.861	0.831	0.692	0.855
1973.2–constant	-0.020	-0.035	-0.012	-0.046	-0.064	-0.030	-0.049	-0.077	-0.032	-0.050	-0.065	-0.030	-0.048	-0.068	-0.034
1973.2 – trend	-0.002	-0.004	-0.001	-0.024	-0.039	-0.013	-0.023	-0.041	-0.009	-0.024	-0.037	-0.015	-0.024	-0.037	-0.013
1972.3-constant				-0.032	-0.064	-0.006	-0.032	-0.070	-0.002	-0.032	-0.056	-0.011	-0.032	-0.056	-0.009
1972.3 – trend				0.023	0.012	0.035	0.023	0.004	0.037	0.023	0.013	0.035	0.023	0.004	0.033
1990.3-constant							-0.014	-0.027	-0.007	-0.022	-0.033	-0.015	-0.022	-0.033	-0.015
1990.3-trend							0.000	-0.001	0.001	-0.000	-0.001	0.000	-0.000	-0.001	0.000
1979.3–constant										-0.024	-0.035	-0.019	-0.054	-0.070	-0.042
1979.3 – trend										0.000	-0.000	0.001	-0.044	-0.073	-0.024
1979.1–constant													-0.056	-0.107	-0.018
1979 1–trend													0.045	0.023	0.063

Table 1B:
UNITED KINGDOM - Marginal 2 (real interest rate)

	Step 1			Step 2			Step 3			Step 4			Step 5		
	Est. value	Bootstrapped 95% confidence interval		Est. value	Bootstrapped 95% confidence interval		Est. value	Bootstrapped 95% confidence interval		Est. value	Bootstrapped 95% confidence interval		Est. value	Bootstrapped 95% confidence interval	
Constant	0.963	-0.344	2.705	-1.685	-3.728	-0.438	-1.781	-4.227	-0.315	-1.829	-4.288	-0.387	-1.842	-3.793	-0.701
Trend	-0.163	-0.361	-0.094	0.158	-0.026	0.305	0.160	0.042	0.340	0.161	0.047	0.306	0.161	0.037	0.273
AR(1)	0.893	0.745	0.905	0.850	0.726	0.872	0.746	0.531	0.796	0.695	0.436	0.742	0.681	0.492	0.727
1975 3-constant	3.488	2.216	6.441	6.538	4.105	9.404	7.282	4.755	9.903	3.594	-0.903	6.081	3.467	-0.078	5.240
1975 3-Trend	0.161	0.074	0.333	0.304	-0.693	1.232	0.776	0.170	2.098	2.445	1.610	4.462	2.563	1.512	4.693
1974 2-constant				-6.012	-10.41	-3.053	-5.115	-7.946	-1.718	-4.669	-7.089	-1.229	-4.548	-7.247	-1.325
1974 2-Trend				-0.458	-1.900	0.421	-0.951	-2.610	-0.407	-1.197	-2.757	-0.610	-1.263	-2.893	-0.671
1981 2-constant							2.653	1.585	4.697	2.400	1.038	5.080	2.356	0.313	5.636
1981 2-Trend							-0.003	-0.237	0.119	-0.135	-0.302	0.001	-0.490	-0.885	-0.046
1977 1-constant										-5.811	-8.117	-3.359	-8.134	-10.65	-5.719
1977 1-Trend										-1.292	-2.628	-0.712	-0.810	-2.155	-0.388
1979 3-constant													-5.363	-7.349	-2.878
1979 3-Trend													-0.180	-0.753	0.324

Table 2A
UNITED KINGDOM- Marginal 1 log (real income)

Time Period	Dominant Break
1969:3-1996:4	1973:2
1973:2-1996:4	1990:3
1973:2-1990:3	1979:3
1979:3-1990:3	1988:4

Table 2B
UNITED KINGDOM- Marginal 2 (real interest rate)

Time period	Dominant break
1969:3-1996:4	1975:3
1969:3-1975:3	1974:2
1975:3-1996:4	1981:2
1975:3-1981:2	1977:1

4. Stable Marginal Models

The results from the previous section are utilised to model the marginal process for the real income in the United Kingdom given as Table 3A below. Note that in this table (and the ones that follow) the notation s1973p2 etc. represents a step dummy variable at the year and quarter given while tr1973p2 represents a trend-break dummy.

Several points are of interest in considering the specification for real income reported in Table 3A. First, evidence of instability in the neighbourhood of the dominant break, detected by the procedure which includes breaks one by one, can be interpreted as modelling the *evolution* of the structural change.

Second, the evidence for a unit root in real income is fairly muted. This obviously harks back to Perron's (1989) observation that failure to take account of structural breaks in time series vitiated unit root inferences. In an important sense, our paper is an extension of Perron's original problem, applied to the case of multiple breaks with breaks in mean and trend being allowed. When the breaks are more numerous, clearly the unit root inferential problem is more acute. Similar observations apply to the remaining processes as the results below show.[5] Our particular contribution is to develop a simple method for judging significance of the coefficient estimates in regression models of general form.

Thirdly, it is possible that the inclusion of a richer lag structure would help to eliminate some of the deterministic dummies included in the model. However, this would simply imply that the bootstrapped model would be somewhat more elaborate but identical principles would apply.

Finally, it is of interest to compare the confidence intervals for the variables generated by the bootstrapping programme (at say Step 5, as reported in Tables 1A) with the *p*-values reported in Table 3A. This is in order to obtain an idea of the level of significance of the coefficient estimates as computed using finite sample and asymptotic (or normal) distributions respectively. An inspection shows that our bootstrapping results are easily interpretable in the context of such a comparison and computation of the bootstrapped critical values would confirm this fact formally.

Table 3B below repeats the modelling exercise for the real interest rate. Note that the final step of the bootstrapping regressions, as reported in Tables 1A and 1B corresponds directly to the final form of the models estimated and reported in Table 3A and 3B. The congruence of these latter models is taken as evidence in favour of our claim that bootstrapping algorithms of the form proposed in our paper do have the desired inferential value. Diagnostic testing based on forecast encompassing tests using the final forms of the models show good properties, unless the forecasting horizon is chosen to start at time periods immediately abutting a break date as identified by the above procedure. From this we conclude that our methods are good at detecting breaks within sample but breaks out of sample pose more difficult problems. For consideration of the latter issue, papers by Clements and Hendry (1996.), Chu, Stinchcombe and White (1996) and Banerjee and Hendry (1998) *inter alia* are relevant.

[5] We have currently assumed that the data does not contain unit roots in order to justify bootstrapping without the value of the lag coefficient (or sum of lag coefficients) imposed to one. Basawa *et al.* (1991) have shown that in the presence of stochastic trends, consistency of the bootstrap estimation methods requires *imposition* of the unit root. We will return to this issue in developments of this work. For the purposes of this paper we take the apparent economic interpretability of the paper as providing *prima facie* evidence for the satisfactory operation of our methods.

Table 3A
UNITED KINGDOM - Marginal 1: log(real income)

Modelling yuk(t) by OLS
The present sample is: 1969 (4) to 1996 (4)

Variable	Coefficient	Std.Error	*t*-value	*t*-prob.	PartR^2
Constant	0.98788	0.18505	5.338	0.0000	0.2289
Trend	0.0016359	0.0007568	2.161	0.0332	0.0464
yuk(t-1)	0.83073	0.0317422	6.171	0.0000	0.8771
s1973p2	-0.048462	0.0078404	-6.181	0.0000	0.2847
tr1973p2	-0.023945	0.0054870	-4.364	0.0000	0.1655
s1972p3	-0.031841	0.012608	-2.525	0.0132	0.0623
tr1972p3	0.023478	0.0055279	4.247	0.0001	0.1582
s1990p3	-0.022077	0.0040310	-5.477	0.0000	0.2381
tr1990p3	0.0003414	0.0002416	-1.413	0.1609	0.0204
s1979p3	-0.054406	0.0083268	-6.534	0.0000	0.3078
tr1979p3	-0.044439	0.010956	-4.056	0.0001	0.1463
s1979p1	-0.055732	0.017632	-3.161	0.0021	0.0943
tr1979p1	0.044820	0.010955	4.091	0.0001	0.1485

R^2 = 0.99799; F(12, 96) = 3971.5 [0.0000]; σ =0 .00774206 DW=2.07
AR 1- 5F(5, 91) = 1.744 [0.1325]
ARCH 4 F(4, 88) = 2.2814 [0.0668]
Normality Chi^2(2) = .7644 [0.0923]
Xi*Xj F(29, 66) = 1.0794 [0.3885]
RESET F(1, 95) = 0.079269 [0.7789]

Table 3B
UNITED KINGDOM - Marginal 2: real interest rate

Modelling iuk(t) by OLS
The present sample is: 1969 (4) to 1996 (4)

Variable	Coefficient	Std.Error	t-value	t-prob	PartR^2
Constant	-1.8422	0.67450	-2.731	0.0075	0.0721
Trend	0.16135	0.057442	2.809	0.0060	0.0759
iuk(t-1)	0.68064	0.0546171	12.462	0.0000	0.6180
s1975p3	3.4675	1.6089	2.155	0.0337	0.0461
s1974p2	-4.5480	1.5191	-2.994	0.0035	0.0854
tr1974p2	-1.2633	0.48018	-2.631	0.0099	0.0672
s1981p2	2.3559	1.0712	2.199	0.0303	0.0480
tr1981p2	-0.48976	0.24145	-2.028	0.0453	0.0411
s1977p1	-8.1336	1.2685	-6.412	0.0000	0.2999
tr1977p1	-0.80975	0.38157	-2.122	0.0364	0.0448
s1979p3	-5.3626	1.3026	-4.117	0.0001	0.1501
tr1979p3	-0.17982	0.28117	-0.640	0.5240	0.0042

$R^2 = 0.954065$; F(12, 96) = 166.16 [0.0000]; $\sigma = 1.26409$; DW = 2.02
AR 1- 5F(5, 91) = 0.88856 [0.4922]
ARCH 4 F(4, 88) = 0.54719 [0.7015]
Normality Chi^2(2) = 0.89038 [0.6407]
Xi^2 F(19, 76) = 1.5192 [0.1029]
Xi*Xj F(34, 61) = 1.2821 [0.1966]
RESET F(1, 95) = 0.013735 [0.9070]

5. Stable Conditional Model

Following Banerjee and Urga (1995, 1996), the next step in the investigations is to use the break-date information derived from the marginal processes, and to impose the breaks in the conditional models (for log of real money for the UK). We then proceed to bootstrap the conditional models with the imposed break dates in order to discover any further distinct breaks in the conditional models.

Table 4
UNITED KINGDOM – Conditional: log(real money)

Modelling m-puk by OLS
The present sample is: 1969 (4) to 1996 (4)

Variable	Coefficient	Std.Error	*t*-value	*t*-prob	PartR^2
Constant	1.9958	0.28507	7.001	0.0000	0.3380
Trend	-0.0045529	0.0012433	-3.662	0.0004	0.1226
m-puk(t-1)	0.59212	0.0565691	0.467	0.0000	0.5330
yuk(t-1)	0.37331	0.088466	4.220	0.0001	0.1565
iuk(t-1)	-0.000867	0.0003984	-2.178	0.0318	0.0471
tr1975p3	0.018744	0.0030363	6.173	0.0000	0.2842
tr1974p2	-0.017948	0.0036029	-4.982	0.0000	0.2054
s1977p1	-0.022174	0.0095813	-2.314	0.0228	0.0528
s1979p3	-0.055607	0.0068390	-8.131	0.0000	0.4078
tr1979p3	-0.0044354	0.0009822	-4.516	0.0000	0.1752
tr1972p3	0.0049059	0.0020993	2.337	0.0215	0.0538
s1990p3	-0.022211	0.0053058	-4.186	0.0001	0.1544
tr1990p3	0.0041908	0.0007362	5.692	0.0000	0.2523

R^2 = 0.996421; F(12, 96) = 2227.4 [0.0000]; σ = 0.00986951; DW = 1.87
AR 1- 5F(5, 91) = 1.3721 [0.2422]
ARCH 4 F(4, 88) = 0.6653 [0.6178]
Normality Chi^2(2) = 3.048 [0.2178]
Xi^2 F(21, 74) = 1.2008 [0.2762]
RESET F(1, 95) = 2.2961 [0.1330]

Table 4 reports the model that results for UK. Our method confirms the results reported in Caporale *et al.* (1997)). No additional dummy variables, *i.e.* not directly derived from the marginal models, turn out to be necessary. A sufficient set of dummies is needed to achieve a well-specified equation, in particular the inclusion of the dummies is necessary to achieve normality of the residuals. Moreover, these models are parsimonious in that only a minimum set of dummies from the marginal processes is significant. Thus, there is evidence of presence of co-breaking (Hendry, 1996), such that some of the breaks of the marginal processes are eliminated in the conditional model. Finally, the main conclusion that we may draw from the conditional model is that we are far away from a stable relationship between money, income and interest rate in the absence of stabilising dummies. Further, the homogeneity restriction on income, claimed by Hoffman *et al.* (1995) and (1996), is rejected in the case of UK.

6. Conclusions

Based on previous work by us, we show how finite sample bootstrapping methods can help to detect multiple breaks in systems of equations with long time series. The method of Banerjee and Urga (1995, 1996), where *single* break date findings in the marginal models are imposed in the conditional model and then the conditional model estimated, is extended to cover the case of multiple (≥ 2) breaks in marginal and conditional models by using the technique of dominant break dating. A detailed empirical investigation of a small monetary system for the United Kingdom establishes the viability of our methods in developing structurally stable and dynamically well specified dynamic regression models. Our empirical exercises provide further evidence of the standard result that money demand functions exhibit instability.

Data Appendix

UK data are taken from the IFS-Datastream database. Money demand is expressed as a seasonally adjusted M0 aggregate (We use M0 and not M1 in light of the fact that M1 is not clearly defined for the period before 1980 and is not recorded from the early 1990's onwards.) Income is measured as GDP (1990 prices), seasonally adjusted. Interest rate is defined as the three-month Treasury bond rate. The latter two series are the same as in the Hoffman and Rasche (1995) data set but updated by us.

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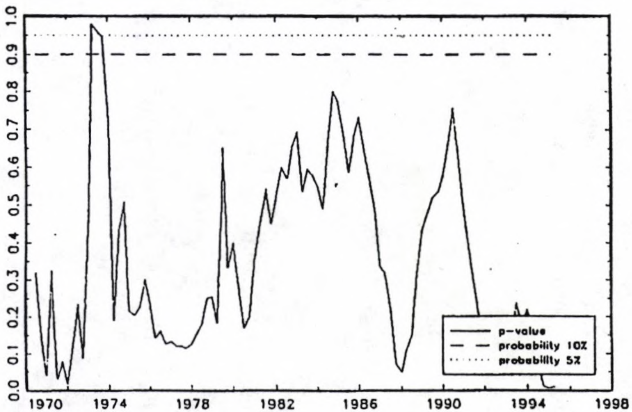
STEP 1: UK – real income, 1969:3–1996:4
max p–value breakpoint: 1973:2



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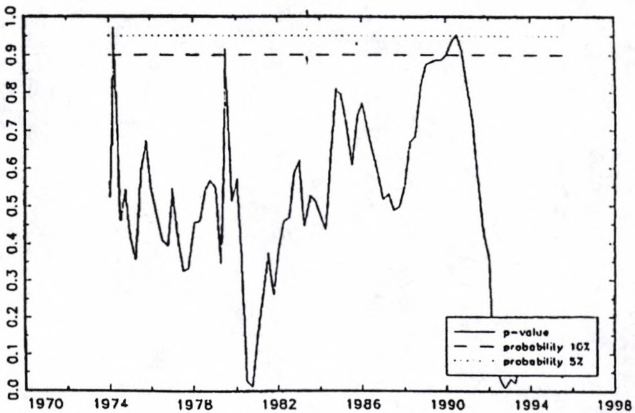
max p–value breakpoint: 1990:3



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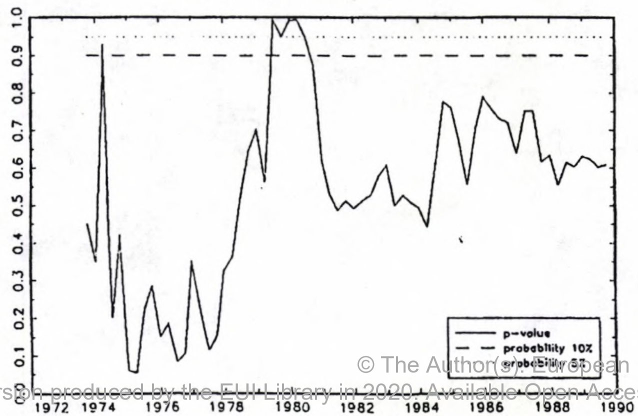
max p–value breakpoint: 1979:3



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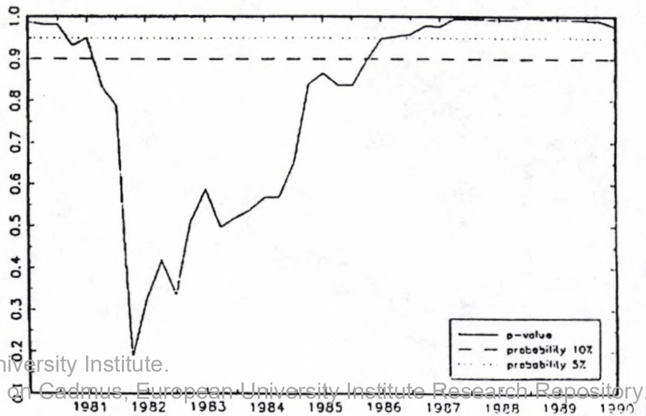

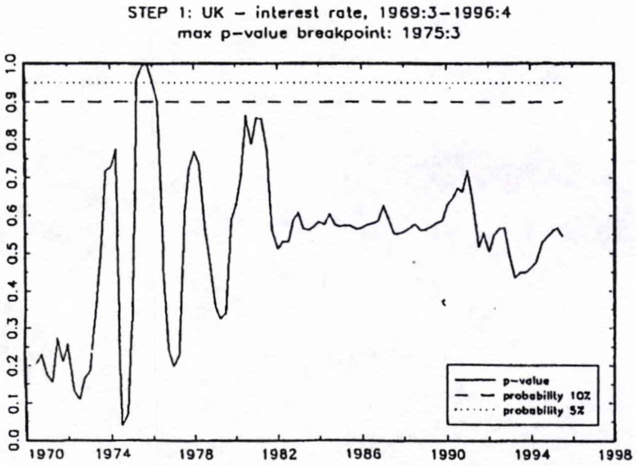
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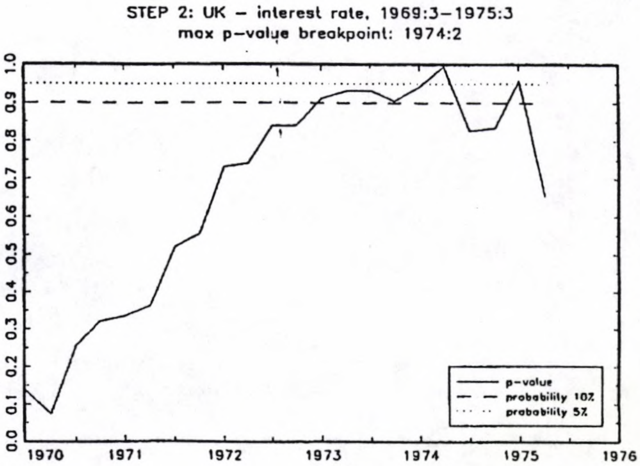
probability 5%
1970
1974
1978
1982
1986
1990
1994
1998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probability 5%
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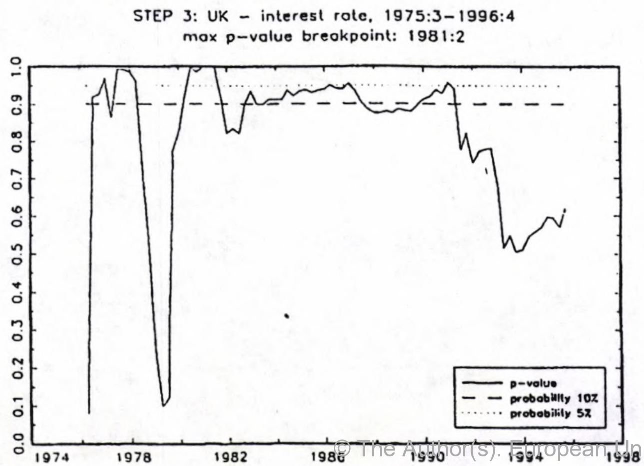
1971
1972
1973
1974
1975
1976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probability 5%
1974
1978
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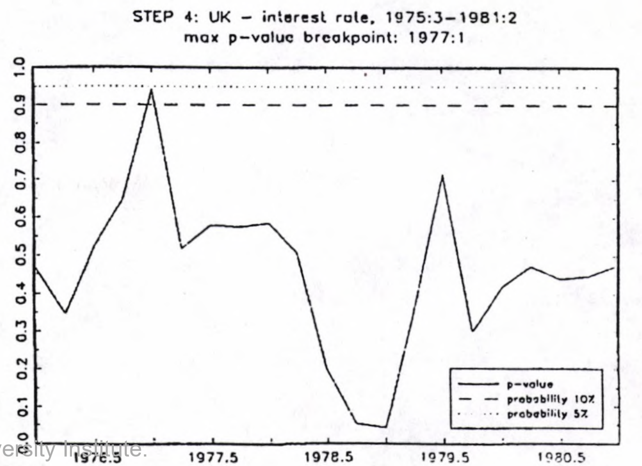
1986
1990
1994
1998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max p-value breakpoint: 1977:1
p-value
probability 10%
probability 5%
1976.5
1977.5
1978.5
1979.5
1980.5

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